Description of the lesson series
Teacher's Guide

Title	*Integers and their addition and subtraction using tokens*
Time	*4-6 school hours (depending on students' tempo and learning level)*
Grade	*Grades 5-8 (students 11-15 years old)*
Aim of the lesson cycle and its brief description	*The aim of this series of lessons is to shape the concept of negative numbers alongside addition and subtraction operations on integers using tokens.* *The scenario can be used both in earlier grades to introduce integers and in later grades for repetition.* *As students play with the concrete model (tokens), they build the concept of a negative number as the opposite of a given positive number and develop an understanding of the operation of addition as putting in tokens and subtraction as taking away tokens.* *As a result, students undertake mathematical modelling.* *This approach to shaping negative numbers is not based on the use of order in a set of integers (we do not compare a negative number to zero), it is an algebraic model that can be extended to shape the concept of an algebraic expression and its opposite, as well as the addition and subtraction of such expressions.*
Teaching materials	*Each student is given 10 round tokens of each colour (white/black) to use as tools during the lessons.*

A linguistic note on working with tokens in the context of integers and algebraic expressions:

In our scenarios, we are careful to keep the two worlds - the world of mathematics, i.e. abstractions, and the world of real objects - in our case tokens - linguistically separate. Thus, in the context of tokens, we use terms that describe their appearance: white/black round/ oblong/square token rather than the short-form white circle/rectangle/square. Similarly, in the context of tokens, we mention placing and taking away tokens – while in the context of mathematics, we discuss addition and subtraction operations. We also make a point of verbally reading action signs as add/subtract, rather than just naming them plus/minus signs. We believe that modelling arithmetic and algebraic expressions with clarity and linguistic correctness in mind is of great value and is highly recommended.

PART 3

Part 3

Topic: Addition and subtraction of integers using tokens

ACTIVITY 1: SUBTRACTION – DISCUSSION OF HOMEWORK FROM THE WORKSHEET

[Appendix: A_ENG_Subtraction worksheet]

Discussion of results and advice formulated by students.

If someone includes, for example, **3 - 6 = 3 + (-6)** in their homework assignment, this can provide an introduction to Activity 2.

ACTIVITY 2: OPPOSITE NUMBER - INTRODUCTION OF TERMINOLOGY

If the students are not familiar with the concept of opposite numbers, we conduct a dialogue as in the example below. If they do know - we ask them to recall the definition (the dialogue can be similar to the following).

- We have so far talked about black and white tokens. What was the most important rule regarding the same number of white and black tokens taken together?

> *S: they cancel each other out*

- How can we write this using the numbers 1 and -1?

> *S: 1 + (-1) = 0*

- What could such tokens be called? What are they to each other in this situation? Give your ideas.

> *Students suggest different names - brainstorm.*

- You have provided various interesting suggestions. From now on, we'll agree to call them OPPOSITE TOKENS, because that's what mathematicians have agreed upon:

Two numbers that add up to zero are called OPPOSITE NUMBERS, *just as tokens add up to zero.*

> *- What number is the opposite number to 2?*
>
> *S: -2*
>
> *- Why?*
>
> *S: because 2 added to -2 results in 0*
>
> *- What number is the number opposite to -2?*
>
> *S: 2*
>
> *- Why?*
>
> *S: because -2 added to 2 results in 0*

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- Provide the number opposite to the number -100.

S: 100

- Why?

(....)

[The number of examples is decided by the teacher depending on the need].

-How do a number and its opposite differ in notation?

S: there is a minus before the number notation

-Then let's turn our attention to this notation again. How do we write the number opposite to 2?

S: -2

The mathematicians also agreed that they would write the opposite number to a given number by adding a minus sign before that number. *This has the advantage that the lazy mathematician does not have to calculate in his head what the opposite number to a given number is, he can automatically write such a number by adding a minus sign in front.*

- Then how do we write the number opposite to -2?

> *S: - (-2)*

> If the answer were 2:

> *- And how can we do it differently if, when forming the opposite number, we add a minus at the front?*

> *S: - (-2)*

If the students don't say it, the teacher guides them.

- When I say opposite to, what sign do I add? What does it mean when I have to write the number opposite to -2 in this way?

- Yes, and we know that the number opposite to -2 is what number?

> *S: 2*

- What can we say about the numbers 2 and -(-2)?

> *S: -(-2) = 2*

ACTIVITY 3: SUBTRACTING A NUMBER MEANS ADDING ITS OPPOSITE

Collaborative work:

The teacher writes the formulation and the first equation on the board:

Task: Justify using tokens that:

> **$4 - 5 = 4 + (-5)$**

> **$-7 - 3 = -7 + (-3)$**
> REMARK: Agreement on the notation: -7 - 3 is the same as (-7) - 3.

The teachers discuss together with students; as in Lesson 2.
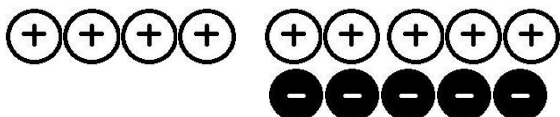
Example run:

- Let's look a thet subtraction 4 - 5. How can we implement it with tokens?
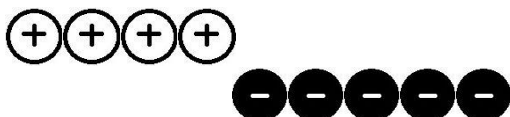
(The teacher waits for the Second Method)

- In order to take some tokens, we can first add them in the form of neutral pairs, so how would we do this here? We can add 5 neutral pairs, because we are supposed to take away 5 white tokens.

We therefore have:

- How do we perform the action – taking away the 5 white tokens, how can we save what's left?

S: **4 + (-5):**

- What have we shown?

S: That 4 - 5 = 4 + (-5).

- So we have justified the equality that instead of subtracting 5 from 4, we can add the number -5 to 4, this means number relative to 5?

S: The opposite

Task: Justify using tokens that: -7 – 3 = -7 + (-3)

- Let's see how it goes in the second example

We ask students to carry out the reasoning themselves. .... So here it is similar... instead of... I can....

- And if I had another example, e.g. .... Do you know what conclusion we would come to at the end?

S: Instead of subtracting we can add the opposite number.

ACTIVITY 3: ADDITION AND SUBTRACTION - COMBINED EXERCISES

[GAME - WHEEL OF FORTUNE]

Option I:

Working in pairs.

If there is an odd number of students in the class, form one trio from the more gifted students (to equalise the time of the pairs as much as possible).

Students in pairs on one phone open the app. They take turns spinning the wheel of fortune. The other student checks the result. If the result is correct, student I receives 1 point, if incorrect, student II (the checker) receives a point, provided he/she can explain the error and convince the student to solve differently. If they can't explain it, no one gets a point and they continue playing.

Depending on the students' calculating ability, 2-3 rounds are conducted.

Option II:

Collaborative work

Students take it in turns to approach the board by spinning the wheel of fortune, which is shown on the computer projector.

Link: https://wordwall.net/resource/64757949

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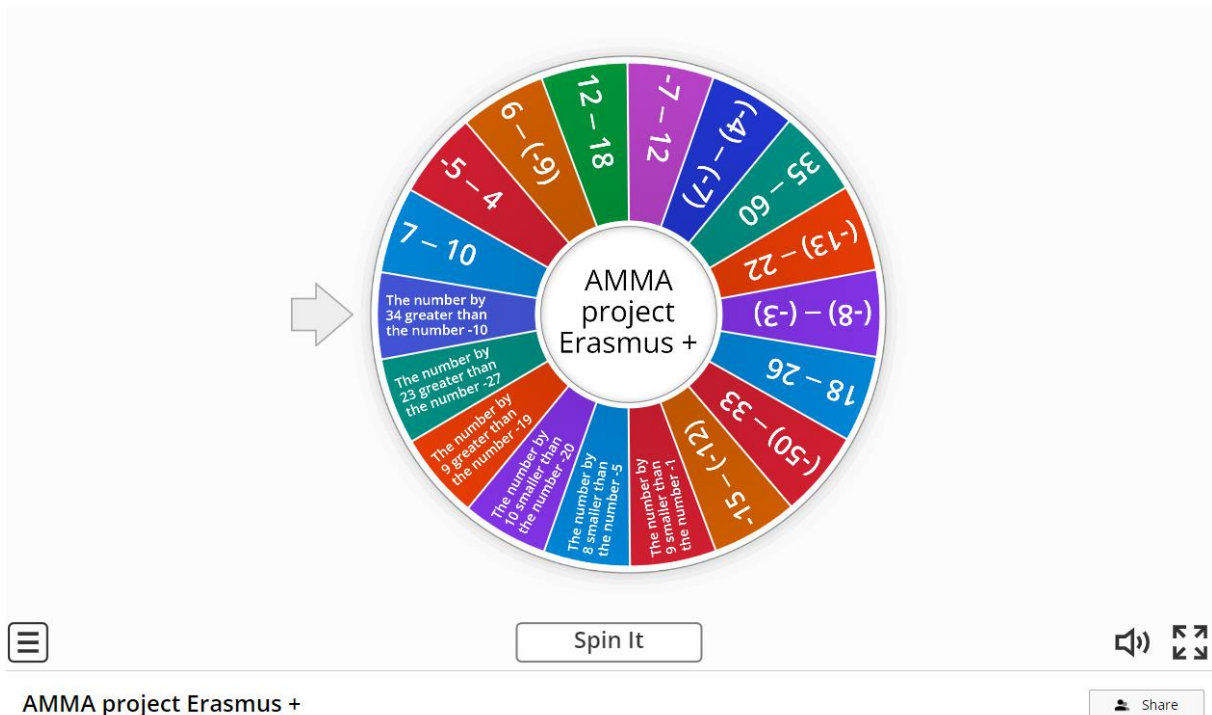
ACTIVITY 4. POST-TEST

[Appendix: A_ENG_Post-test]

Write down the result of the operations. In each case, **explain how you arrived at your result.**

a) $-5 + 3 =$	b) $5 + (-3) =$	c) $-5 + (-3) =$	d) $3 - 5 =$

Write down the result of the operations. In each case, **explain how you arrived at your result.**

e) $-3 - 5 =$	f) $-5 - (-3) =$	g) $-5 - 3 =$	h) $5 - (-3) =$

ACTIVITY 5. SUMMARY – QUESTIONNAIRE

[Appendix: A_ENG_Questionnaire]

1. To what extent has the token method helped you understand and perform integer operations? Mark one answer on a scale of 1 to 5 (where 1 means it has not helped you at all and 5 means it has been very useful).

Has not helped me at all   Has been very useful

2. Did you enjoy the token method? (please circle your answer)

   NO   /   I DON'T KNOW   /   YES

3. What do you think of lessons with tokens? Write your opinion.

Possible additional work for a gifted student who has completed work early

ACTIVITY 6*

Individual work.

Task 1. Calculate. Underline the examples in which the token method has helped you.

      a) $6 + (-4) + (-10) =$
      b) $-4 - (-9) - 3 =$
      c) $-16 + 7 - 28 =$

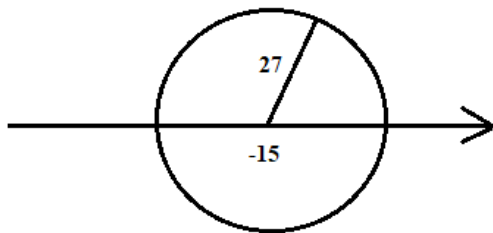Task 2. Calculate. Underline the examples in which the token method has helped you.

      a) $8 - (-9) + (-11) - 5 =$
      b) $-21 + (-7) - 15 - (-32) =$
      c) $-11 - (-6) + (-5 + 8) =$
      d) $-(6 - 9) - (-12 + 15) =$
      e) $16 - (-4 + 7) + (5 - 9) =$
      f) $(-8 + 12 - 3) - (-6 + 13) =$
      g) $-19 - (-18 - 7) + (-34) =$
      h) $-27 + (-26 - 17) + 14 =$

Task 3. Represent the number -1 in the form of:

    a) The difference of two negative numbers
    b) The sum of two numbers with opposite signs
    c) The difference of two numbers of opposite signs
    d*) The sum of two negative numbers

Task 4. This circle intersects the number axis at two points. Provide their coordinates.

a)

b)

This material is provided by the AMMA Team, responsible institution: Pedagogical University of Krakow