Teacher Guide
Description of the lesson series

Title	**Algebraic expressions and their addition and subtraction using tokens**
Time	*5-8 school hours (depending on the students' pace and learning level)*
Grade	*Grades 6-8 (students 12-15 years old) or Grade 9 (for students with difficulties in learning mathematics)*
Aim of the lesson cycle and its brief description	*The aim of this series of lessons is to shape the concept of an algebraic expression and its opposition, as well as the addition and subtraction of such expressions using tokens.* *The scenario can be used both in younger grades as an introduction to algebraic expressions and for repetition lessons with students in older grades.* *As students play with the concrete model (tokens), they build up the concept of the algebraic expression and its opposition, and develop an understanding of the operation of addition as adding tokens, and subtraction as taking away tokens.* *Through this, students undertake mathematical modelling.*
Teaching materials	*Each student is given 10 tokens of each colour (white/black) and each shape: (round/oblong/square), for a total set of 60 tokens, to use as tools during the lessons.*

A linguistic note on working with tokens in the context of integers and algebraic expressions:

In our scenarios, we are careful to keep the two worlds - the world of mathematics, i.e. abstractions, and the world of real objects - in our case tokens - linguistically separate. Thus, in the context of tokens, we use terms that describe their appearance: white/black round/ oblong/square token rather than the short-form white circle/rectangle/square. Similarly, in the context of tokens, we mention placing and taking away tokens – while in the context of mathematics, we discuss addition and subtraction operations. We also make a point of verbally reading action signs as add/subtract, rather than just naming them plus/minus signs. We believe that modelling arithmetic and algebraic expressions with clarity and linguistic correctness in mind is of great value and is highly recommended.

PART 3

Part 3

Topic: Subtracting algebraic expressions using tokens

ACTIVITY 1: Introduction to subtraction

- *What operation will describe the following situation:*

  - *I have 5 white oblong tokens* $\boxed{x}\boxed{x}\boxed{x}\boxed{x}\boxed{x}$ *and I take away 3 of them* $\boxed{x}\boxed{x}\boxed{x}$. *How many do I have left?*
    Students describe: $5x - 3x = 2x$
  - *I have 6 black square tokens* $\boxed{-x^2}\boxed{-x^2}\boxed{-x^2}\boxed{-x^2}\boxed{-x^2}\boxed{-x^2}$
    and I take away one such token $\boxed{-x^2}$. *How many do I have left? How do we write this down?*

Students describe: $-6x^2 - (-x^2) = -5x^2$

AGREEMENT: TAKE AWAY MEANS SUBTRACT

ACTIVITY 2: Subtraction exercise (without adding neutral pairs)

Collaborative work. Discussion.

- *E.g.* $6x - 4x$. *What would this mean on the tokens? Write the operation on the board and represent it with tokens (on the board)*

  (The students perform the modelling, the activity is done on the board – removal of tokens:
  I have 6 white oblong tokens, I take away 4 white oblongs, so I have 2 white oblongs left).

- *And what would the operation look like on the tokens* $(-5x^2) - (-3x^2)$ *?*
  (The students perform the modelling, the activity is done on the board – removal of tokens:
  I have 5 black square chips, I take 3 black square chips, so I have 2 black square tokens left).

Individual work, but summarised with student statements and a note:

\- *Please represent the operation by using tokens and justify the result.*

- $\boldsymbol{-4x - (-x) =}$ **......**

The students arrange the model. They then read out an interpretation of the operation on the model:

- S: I have 4 black oblong tokens and I take away 1 black oblong, so I am left with 3 black oblongs.

(Students cross out the token that was taken away in a note. This notation appears on the board)

- Please write down the following activity in your notebook and draw a justification for the result by using tokens.

- $-2x-(-2x)=$
- $-7x^2-(-3x^2)=$

ACTIVITY 3: Modelling subtraction in two ways (adding different numbers of neutral pairs)

Working together. Dialogue.

We ask students the question:

- How do we illustrate the example $\mathbf{3x^2-6x^2}$ *using tokens? Have we seen something like this before? (Teacher writes the example on the board)*

By asking further questions, we attempt to get the answer that a similar situation existed with negative numbers and round tokens in the case where we had to take away tokens that were not there.

Possible questions:

- We are supposed to take six white square tokens away and we only have three white square tokens, so how can this be done?

Students should work out how to add neutral pairs after their experience with whole numbers. If they don't come up with anything – ask more questions about the rules we have in our token game and how this was done with negative numbers when we did not have enough tokens.

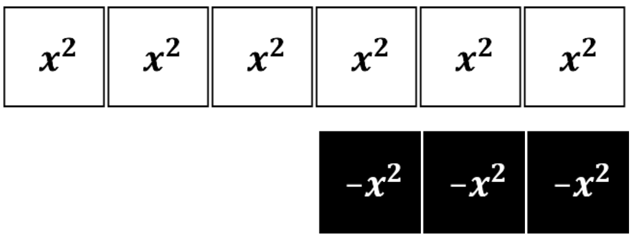
- How many neutral pairs made of square tokens can we place?

One of the following options should appear. After one of them appears (and is discussed), ask how this could be done differently, and discuss the other way.

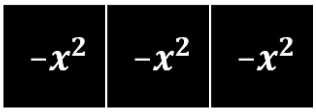
<u>First way</u>

Reasoning:

Since we have to take 6 white square tokens we must first have them, so we add 3 pairs of neutrals:



If we now take away the 6 white square tokens, we get the result straight away:

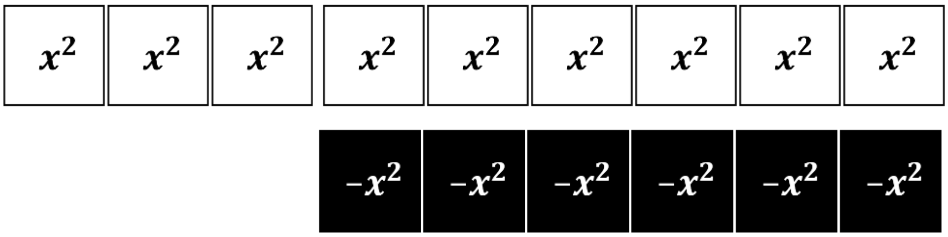


, i.e. $-3x^2$.

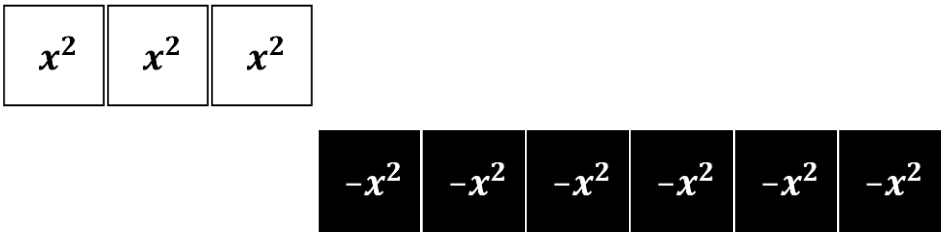
<u>The second way</u>

It can be done the same way we did it with the round tokens: in order to take away tokens, we first have to add them to each other in the form of neutral pairs, so how do we do this here? We immediately add 6 neutral pairs, because we have to take 6 white square tokens.

Then we have:



If we now take away the 6 white square chips, we get the same result as $3x^2 + (-6x^2)$:



That is, $3x^2 - 6x^2 = 3x^2 + (-6x^2)$. *From here, we already know what to do. (Though we now need to model the second operation - addition.)*

Students write down the examples and the result in their notebooks and provide a justification for the result by using the tokens.

ACTIVITY 4: Subtraction - problem regarding the addition of neutral pairs and reminding students that subtraction can be replaced with the addition of the opposite expression

Collaborative work. Talking

- $\boldsymbol{2x - (-x)}$

We pose this question to the students:

- How do we represent the operation $\mathbf{2x} - (-\mathbf{x})$ *by using tokens? Have we seen anything similar before? (Teacher writes the operation on the board)*

By asking further questions, we look for the answer that a similar situation was happened with negative numbers and round tokens, in a situation where we were supposed to take tokens that were not there.

- We are supposed to take one black oblong token, so I must have it. We add the tokens: one white oblong and one black oblong - this way we will be able to take one black oblong. This will leave three white oblongs, i.e. as a result of the $3x$.

- How can this be done differently? What do we already know regarding subtraction?

S: You can $\mathbf{2x} - (-\mathbf{x}) = \mathbf{2x} + \mathbf{x}$

- Justify by using tokens that the result will be the same.

**) Note:*

If the students say that subtracting one black elongated token means adding one white elongated token, we note that this is a very good point and ask them to justify with the tokens that $\mathbf{2x} - (-\mathbf{x}) = \mathbf{2x} + \mathbf{x}$. *The students use the tokens to show that when we add* x *to 2x, the result will be the same as when taking tokens away after adding a neutral pair.*

Working together

- $-\mathbf{2x} - (-\mathbf{3x})$

  - How do we represent the operation $-2x - (-3x)$ *with tokens? (Have we seen anything similar before?)* (The teacher writes the operation on the board)

  (When reading, note the usage of the word subtract: "minus two x subtract minus three x")

  - What kind of tokens do we have? How many are there?

  S: We have 2 black oblong tokens.

  - What should we do?

  S: Take away 3 black oblong tokens.

  - How do we do it?

The teacher listens to the students' ideas and follows the method the students provide – the idea described below in the **First way** or in the **Second way** may emerge.

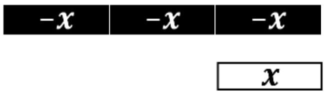
Note: If no ideas emerge, the teacher can continue asking questions such as:

- How do we take away 3 blacks when we only have 2 blacks?

- What do we do to have another black oblong token?

These questions hint towards the **First way.**

First way



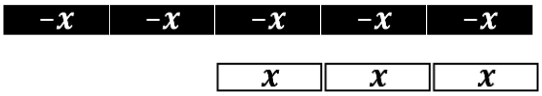
If we now take away 3 black oblong tokens, the result is immediately: x*, i.e. x.*

A second way leading to the discovery of the principle is possible: $-2x-(-3x)=-2x+3x$*, and we already know how to add:*

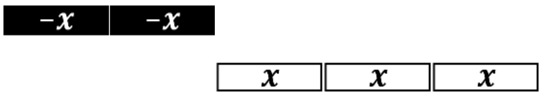
The second way

This can be done the same way as in the previous examples: in order to take away tokens, we first add them to each other in the form of neutral pairs, so how can we do this here? We add 3 neutral pairs, because we are supposed to take away 3 black oblong tokens.

Then we have:



If we now take away the 3 black oblong tokens, we get the same result as $-2x+3x$*:*



That is, $-2x-(-3x)=-2x+3x$*. From here, we already know what to do. (Though we now need to model the second operation, which is addition).*

To summarise:

What do we do to subtract an expression?→ Replace the subtraction with the addition of the opposite expression.

ACTIVITY 5: Modelling subtraction in two ways - justifying subtraction as the addition of the opposite expression - exercises

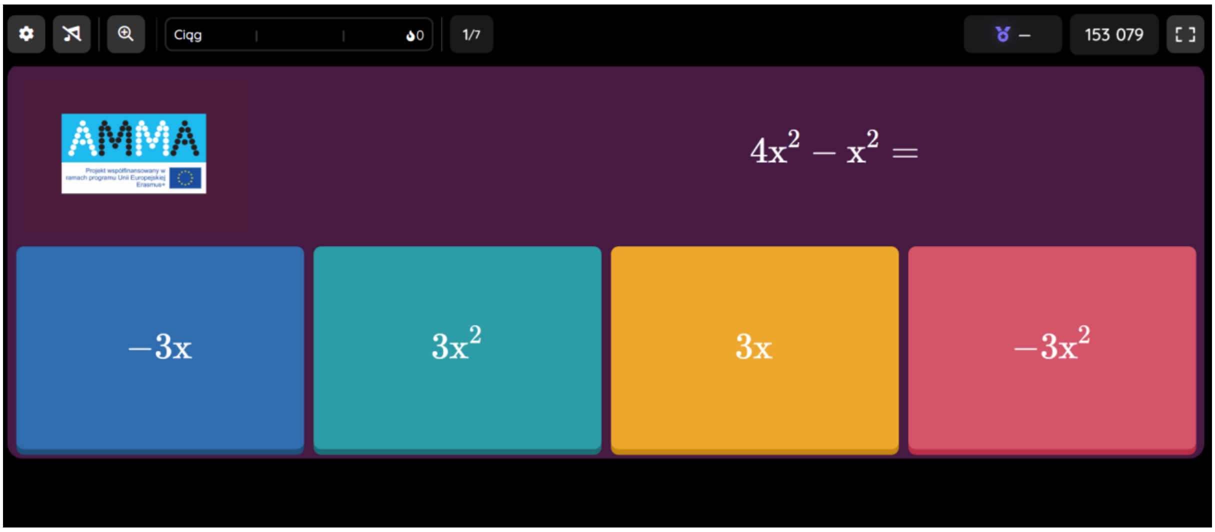
Individual work:

- Please justify the operation given using tokens. Please write the operation with the result in your notebook and draw a justification for the result by using tokens:

- $\mathbf{2x^2}-(\mathbf{-3x^2})=$
- $\mathbf{-4x^2}-(\mathbf{-6x^2})=$

ACTIVITY 6: Game - subtraction exercise

Quizizz:https://quizizz.com/admin/quiz/63d942303aa411001ef9321b?source=quiz_share



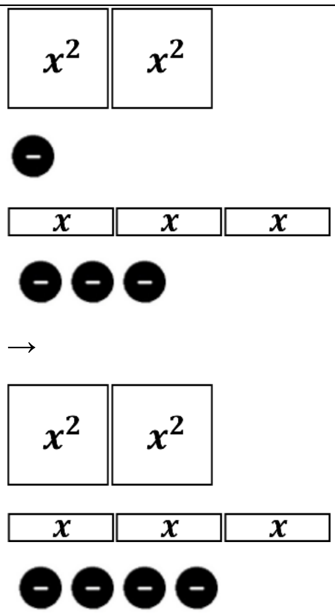
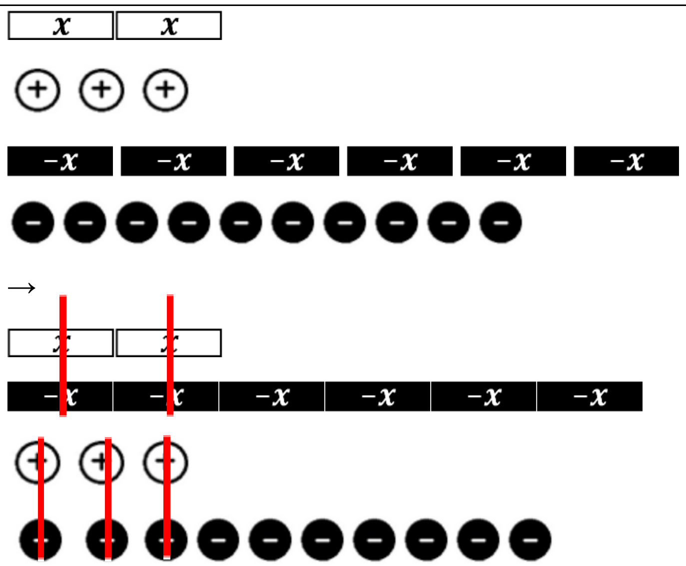
N	Question	Answers			
1.	$4x^2 - x^2 =$	$\mathbf{3x^2}$	$-3x^2$	$3x$	$-3x$
2.	$-3x - (-2x) =$	$\mathbf{-x}$	x	$-5x$	$5x$
3.	$7x - 9x =$	$\mathbf{-2x}$	$2x$	2	-2
4.	$5x^2 - (-2x^2) =$	$\mathbf{7x^2}$	$-7x^2$	$-3x^2$	$3x^2$
5.	$-x^2 - (-9x^2) =$	$\mathbf{8x^2}$	$-8x^2$	$10x^2$	$-10x^2$
6.	$7x - (-1) - (-2x) - 1 =$	$\mathbf{9x}$	$9x - 2$	$5x - 2$	$5x$
7.	$3x - 3x^2 - (-3x) - (-3x^2) =$	$\mathbf{6x}$	$-6x$	0	$6x^2$

ACTIVITY 7: Exercise - simplifying algebraic expressions

Collaborative work - students individually arrange the tokens on their desks, then one person presents them on the board.

We ask:

> *- Has anyone arranged it differently? What do we notice? We aim for the students to notice neutral pairs in the expressions.*

No.	Algebraic expression	Possible model(s)
1.	$2x^2 + (-1) + 3x + (-3) =$	
2.	$2x + 3 - 6x - 10 =$ *(What does it mean to subtract 6x? What did we just say? S: that we can replace this subtraction by the addition of (-6x) i.e. add 6 black oblongs and 10 black round tokens).* *We write down the equivalence:* $2x + 3 - 6x - 10$ $= 2x + 3 + (-6x) + (-10)$	

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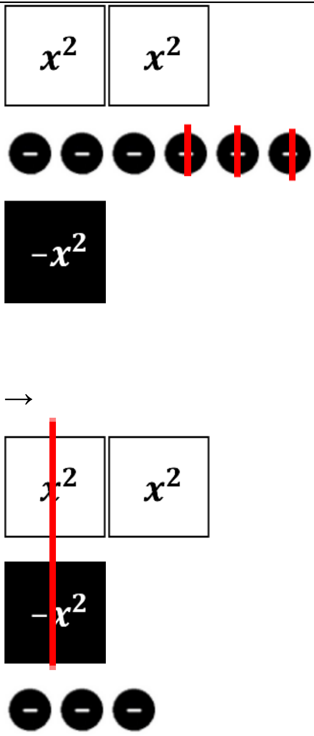
<td>3.</td>
<td>$2x^2 - 6 - x^2 - (-3) =$</td>
<td>
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Individual work:

- *Represent the given expression by using tokens. Simplify it and justify how to do it. Remember to make a drawing.*

- $x^2 + (-x) - 2x + 5 =$