Teacher Guide
Description of the lesson series

Title	**Algebraic expressions and their addition and subtraction using tokens**
Time	*5-8 school hours (depending on the students' pace and learning level)*
Grade	*Grades 6-8 (students 12-15 years old) or Grade 9 (for students with difficulties in learning mathematics)*
Aim of the lesson cycle and its brief description	*The aim of this series of lessons is to shape the concept of an algebraic expression and its opposition, as well as the addition and subtraction of such expressions using tokens.* *The scenario can be used both in younger grades as an introduction to algebraic expressions and for repetition lessons with students in older grades.* *As students play with the concrete model (tokens), they build up the concept of the algebraic expression and its opposition, and develop an understanding of the operation of addition as adding tokens, and subtraction as taking away tokens.* *Through this, students undertake mathematical modelling.*
Teaching materials	*Each student is given 10 tokens of each colour (white/black) and each shape: (round/oblong/square), for a total set of 60 tokens, to use as tools during the lessons.*

A linguistic note on working with tokens in the context of integers and algebraic expressions:

In our scenarios, we are careful to keep the two worlds - the world of mathematics, i.e. abstractions, and the world of real objects - in our case tokens - linguistically separate. Thus, in the context of tokens, we use terms that describe their appearance: white/black round/ oblong/square token rather than the short-form white circle/rectangle/square. Similarly, in the context of tokens, we mention placing and taking away tokens – while in the context of mathematics, we discuss addition and subtraction operations. We also make a point of verbally reading action signs as add/subtract, rather than just naming them plus/minus signs. We believe that modelling arithmetic and algebraic expressions with clarity and linguistic correctness in mind is of great value and is highly recommended.

PART 2

Part 2

Topic: Introduction to addition of algebraic expressions using tokens

ACTIVITY 1: Adding uninomials on tokens *{title for teacher only}*

Collaborative work – the teacher uses large magnetic or virtual tokens - work at the board, record the activity on the side:

- *What operation will describe the situation:*

- *I have got* [x][x] *and* [x][x][x]. *How many do I have together?*
  S: I have five white oblong tokens
  - How do we write this down?
  Students describe: $2x + 3x = 5x$

AGREEMENT: PLACE means ADD

- *I have got one white square token* [x^2] *and I place two white square tokens* [x^2][x^2]. *How many do I have together?*
  S: I have three white square
  - How do we write this down?
  Students describe: $x^2 + 2x^2 = 3x^2$

- *I have got two black oblong tokens* [$-x$][$-x$] *and I put four black oblong tokens* [$-x$][$-x$][$-x$][$-x$]*.How many do I have together?*
  S: I have six black oblong tokens.
  - How do we write this situation down?
  Students write down: $-2x + (-4x) = -6x$

- *I have got one black square token* [$-x^2$] *and I put three black square tokens* [$-x^2$][$-x^2$][$-x^2$]. *How many do I have together?*
  S: I have four black square tokens.
  - How do we write this situation down?
  Students write down: $-x^2 + (-3x^2) = -4x^2$

NOTE ON WRITING – we agree that if we are talking about an opposite expression (with a minus sign), we write it in brackets. With the first component of the sum, we do not need to use brackets.

- *Next to the three white oblong tokens* x x x *I place one black oblong token* $-x$.
  Students: together we have two white oblong tokens and write them down as:
  $3x + (-x) = 2x$
- *Next to* $-x^2$ $-x^2$ $-x^2$ *I place* x^2 x^2
  Students answer: together I have one black square token and write it down as:
  $(-3x^2) + 2x^2 = -x^2$

ACTIVITY 2: Mathematizing the polynomial model without using neutral pairs *{title for teacher only}.*

- *How many tokens do I have in total:*

\- *To two white square tokens* x^2 x^2 *I add three white round tokens* ⊕ ⊕ ⊕ *and one black square token* $-x^2$

- *What expression does this set of tokens represent?*

Students answer: together I have the following tokens: one white square and three white round ones and write them down as: $2x^2 + 3 + (-x^2) = x^2 + 3$

The teacher shows the students which tokens they have to arrange. The students **independently arrange them on their desks and write down the expression.**

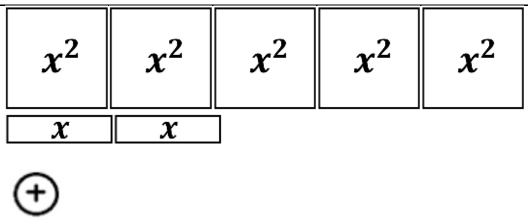
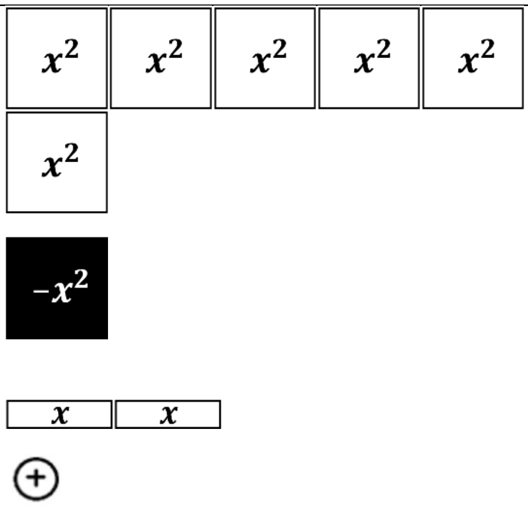
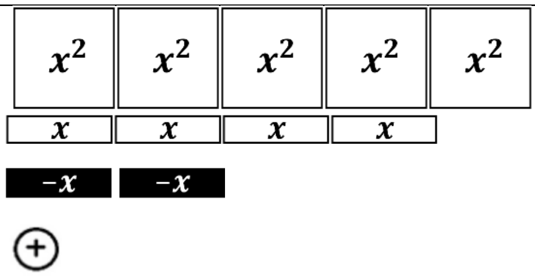
T	The teacher places these tokens on the board	Students respond	*Notes for the teacher* / Issues for discussion
1.	x x ⊕	$2x + 1$	
2.	$-x^2$ ⊖ ⊖	$-x^2 + (-2)$	*NOTES:*
3.	x^2 $-x$	$x^2 + (-x)$	*- Students write algebraic expressions in their notebooks under each numbered case:*
4.	$-x^2$ x ⊖	$-x^2 + x + (-1)$	*1.* $2x + 1$ *2.* $-x^2 + (-2)$ 3....
5.	$-x^2$ $-x^2$ x	$-2x^2 + x$	

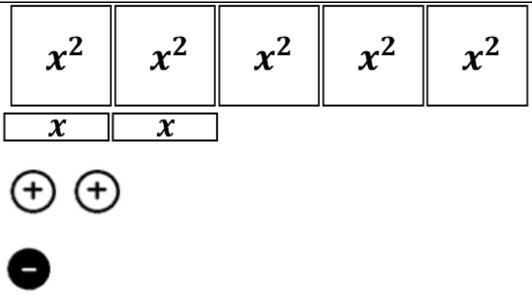
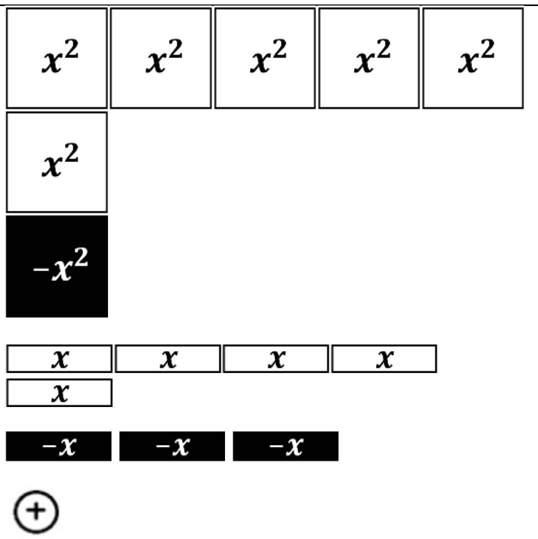
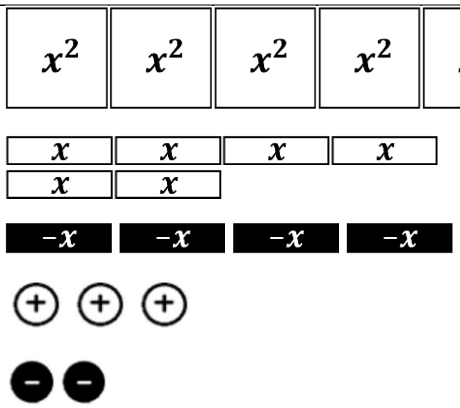
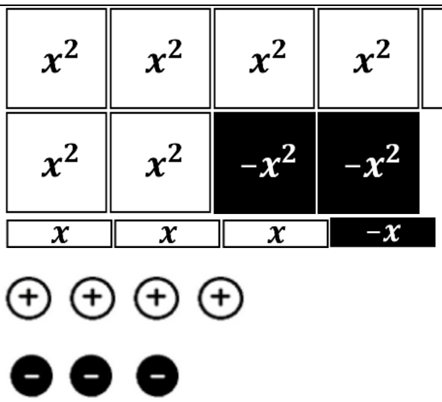
We check the correctness in the writing of successive algebraic expressions (e.g. the teacher reads the number of the situation depicted with the model and the students read the algebraic expression)

ACTIVITY 3: Mathematising a polynomial model using neutral pairs

The teacher tells the students which tokens to arrange (he/she can arrange them himself/herself at the same time). The students ***arrange them independently on their desks****. They then answer and justify which algebraic expression is shown.*

A discussion can be held on the most effective way of stacking.

No.	The teacher tells the students to take the following tokens:	Students arrange the tokens	Students' response	*Notes for the teacher/ Issues for our discussion*
1.	5 white square, 2 white oblong, and 1 white round		$5x^2 + 2x + 1$	*- Students place them however they want on the desk; if some place it chaotically and others neatly, one below the other, then there is an opportunity to discuss the approach–s - which is more useful*
2.	6 white square, 1 black square, 2 white oblong, and 1 white round		$6x^2 + (-x^2) + 2x + 1$ or $5x^2 + 2x + 1$	*- If ordering (stacking white over black or vice versa of figures of the same shape) does not come out naturally and spontaneously from the students, deliberately ask questions about the most effective way of stacking. Is it enough to simply place white over black or vice versa?*
3.	5 white square, 4 white oblong, 2 black oblong, and 1 white round		$5x^2 + 4x + (-2x) + 1$ or $5x^2 + 2x + 1$	

4.	5 white squares, 2 white oblongs, 2 white rounds, and 1 black round		$5x^2 + 2x + 2 + (-1)$ or $5x^2 + 2x + 1$	
5.	6 white squares, 1 black square, 5 white oblongs, 3 black oblongs, and 1 white round		$6x^2 + (-x^2) + 5x$ $+(-3x) + 1$ or $5x^2 + 2x + 1$	
6.	5 white squares, 6 white oblongs, 4 black oblongs, 3 white rounds, and 2 black rounds		$5x^2 + 6x + (-4x)$ $+3 + (-2)$ or $5x^2 + 2x + 1$	
7.	7 white squares, 2 black squares, 3 white oblongs, 1 black oblong, 4 white rounds, and 3 black rounds		$7x^2 + (-2x^2) + 3x$ $+(-x) + 4 + (-3)$ or $5x^2 + 2x + 1$	

ACTIVITY 4: Representation of algebraic expressions on a model

Students work individually.

Perform the task in the form of a "rare case" competition: display the given expression using tokens so that no one else the same representation as you. The rarest solution wins. (Scoring: everyone gets as many points as the number of students with that solution. The student with the fewest points wins.)

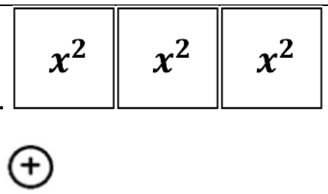
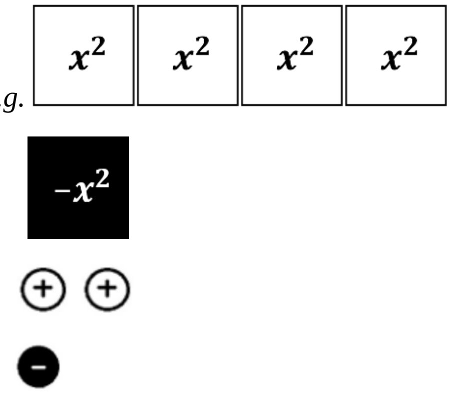
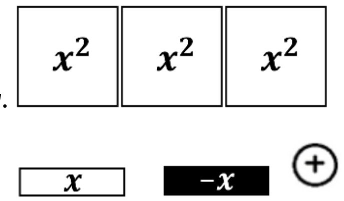
- *Build an interesting set representing an algebraic expression:*

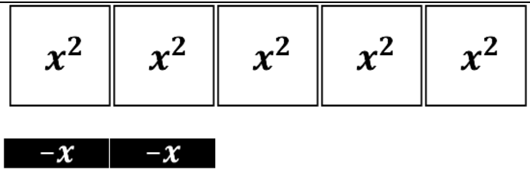
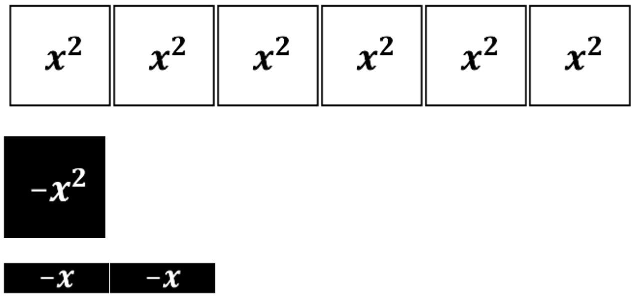
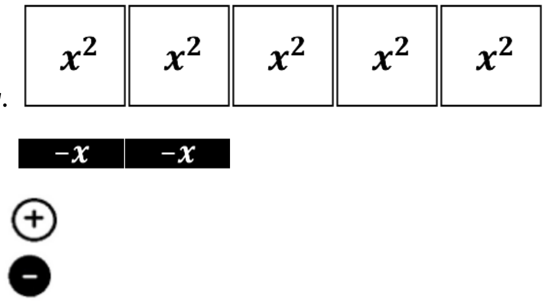
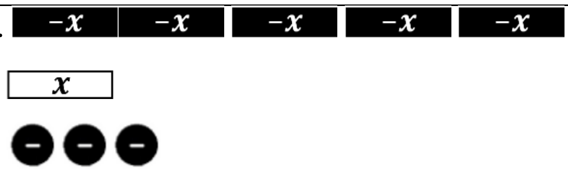
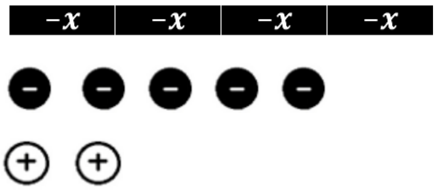
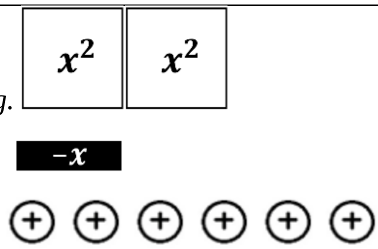
1. $3x^2 + 1$,
2. $5x^2 + (-2x)$,
3. $-4x + (-3)$,
4. $2x^2 + (-x) + 6$,
5. 0

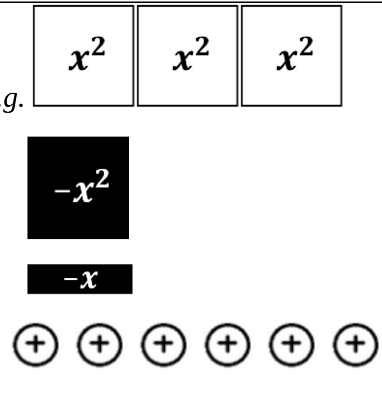
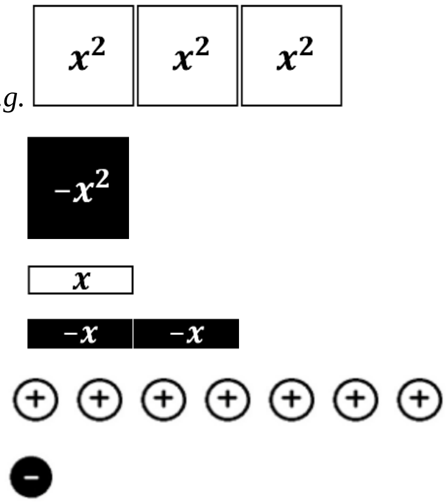
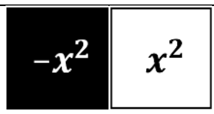
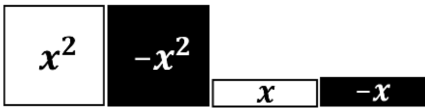
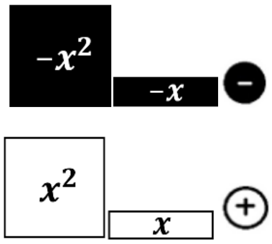
We collect further ideas from the classroom:

- *Has anyone arranged it differently?*

{The table only illustrates expected/selected sample results}.

No.	Algebraic expression	Possible model(s)
1.	$3x^2 + 1$	*e.g.*  *e.g.*  *e.g.* 

2.	$5x^2 + (-2x)$	*e.g.*  *e.g.*  *e.g.* 
3.	$-4x + (-3)$	*e.g.*  *e.g.* 
4.	$2x^2 + (-x) + 6$	*e.g.* 

		e.g.  e.g. 
5.	0	e.g.  e.g.  e.g. 

ACTIVITY 5: Introducing the concept of *OPPOSITE EXPRESSIONS*

Working together*:*

- *E.g.* $2x + 3x$ *What would this mean on the tokens?*
  {Students give a verbal description}

- *And what would the operation look like on the tokens* $(-2x) + (-3x)$*?*
  {Students give a verbal description: I take these tokens: 2 black oblongs and I add 3 black oblongs – in total, I have 5 black oblongs}

- *And* $4x^2 + (-4x^2)$*?*
  {S: returns 0, because the tokens: 4 white square and 4 black square cancel each other out}

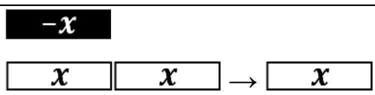
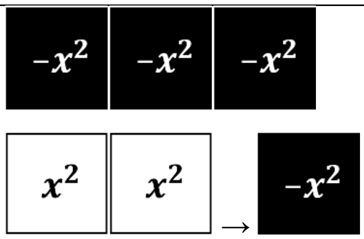
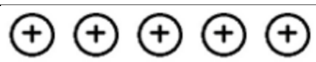
- *Since the addition of these two expressions results in 0, what do we call these two expressions:* $4x^2 + (-4x^2)$*?*
  {OPPOSITE EXPRESSIONS}

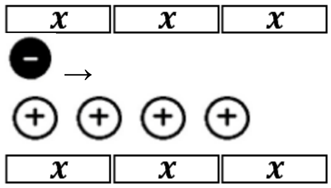
- *Give an example of other opposite expressions*

Working in pairs:

Examples A and B can be represented by students in pairs by using tokens: one person does operation $-x + 2x$*, the other* $2x + (-x)$*. Further examples can be done together.*

- *Please justify the result presented on the tokens.*

Operation	**Students arrange the model and justify**	***Notes for the teacher***
A) $-x + 2x =$ *(first person, on the desk)* $2x + (-x) =$ *(second person, on the desk)* *We do further examples similarly until the students themselves notice the alternation.*		NOTES: - *We ask students to speak, the language of justification is important: if the tokens are the same shape, the white token with the black one will cancel out with white remaining.*
B) $-3x^2 + 2x^2 =$ $2x^2 + (-3x^2) =$		- *The students should notice that the records of the two operations (alternation) can be represented by the same arrangement of tokens, as it is a matter of counting in the correct order. If they do not notice, ask an appropriate question, e.g. after the first four operations: What do you notice?*
C) $5 + 3x + (-1) =$		

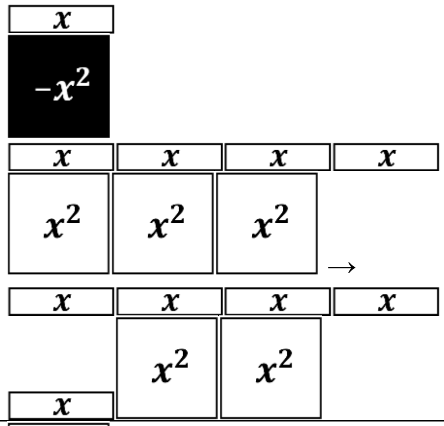


- *Noting to write brackets if there are two signs in a row*

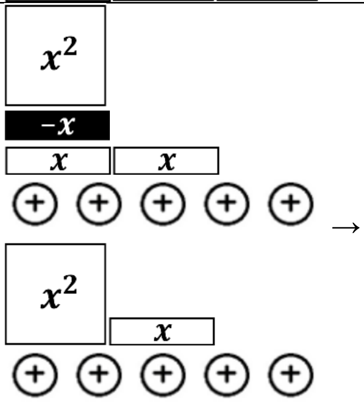
- *Be careful to read the operation character as an addition operation (rather than a plus symbol):*

minus $\mathbf{3x^2}$ ***ADD*** $\mathbf{2x^2}$

D) $x + (-x^2) + 4x + 3x^2 =$



E) $x^2 + (-x) + 2x + 5 =$



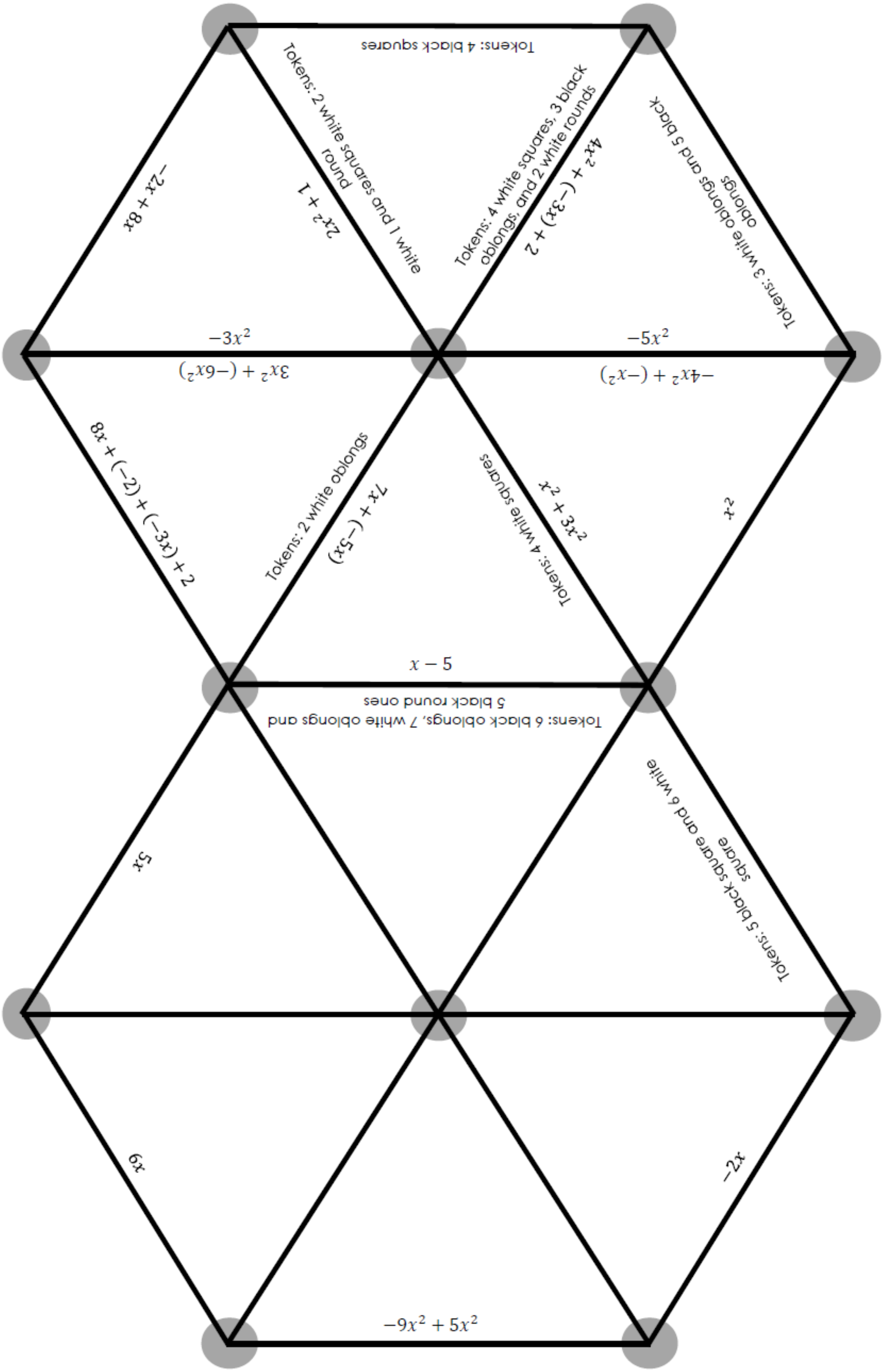
These examples are followed by **a note.**

- Write the chosen activity in your notebook and justify it by illustrating how the result is formed from the corresponding tokens.

ACTIVITY 6: *Game – Trimino*

Students, paired up, are given triangle-shaped pieces (printable cutout of the figure below). Students are to arrange them to match the algebraic expressions and description to the models found on the adjacent edges of the pieces. The reward will be a star for the students.

Depending on the class and its needs, students can be rewarded for the fastest arrangement with a conventional prize.



[Appendix: B_ENG_Trimino]

The table below shows which descriptions should be edge-to-edge (responses to the teacher).

1.	$2x^2 + 1$	Tokens: 2 white squares and 1 white round
2.	$4x^2 + (-3x) + 2$	Tokens: 4 white squares, 3 black oblongs, and 2 white rounds
3.	$-4x^2 + (-x^2)$	$-5x^2$
4.	$x^2 + 3x^2$	Tokens: 4 white squares
5.	$7x + (-5x)$	Tokens: 2 white oblongs
6.	$3x^2 + (-6x^2)$	$-3x^2$
7.	$-2x + 8x$	$6x$
8.	$-9x^2 + 5x^2$	Tokens: 4 black squares
9.	Tokens: 3 white oblongs and 5 black oblongs	$-2x$
10.	Tokens: 5 black square and 6 white square	x^2
11.	Tokens: 6 black oblongs, 7 white oblongs and 5 black round ones	$x - 5$
12.	$8x + (-2) + (-3x) + 2$	$5x$